Solving Linear Systems by Elimination

You can add or subtract the equations in a linear system to obtain a new equation in one variable. This method is called elimination.

Example One: *Use addition or subtraction to eliminate a variable.*

ADD: coefficients are opposites

$$\begin{aligned} 3x - 2y &= 1 \\ + \quad 2x + 2y &= 14 \\ \hline 5x &= 15 \\ \frac{5x}{5} &= \frac{15}{5} \\ \boxed{x = 3} \end{aligned}$$

Substitute:

$$\begin{aligned} 2x + 2y &= 14 \\ 2(3) + 2y &= 14 \\ 6 + 2y &= 14 \\ -6 \quad &\quad -6 \\ 2y &= 8 \\ \boxed{y = 4} \end{aligned}$$

Solution: (3, 4)

SUBTRACT: Coefficients are the same

$$\begin{aligned} 3x - 2y &= -2 \\ - \quad 3x + y &= 10 \\ \hline -3y &= -12 \\ \frac{-3y}{-3} &= \frac{-12}{-3} \\ \boxed{y = 4} \end{aligned}$$

Substitute:

$$\begin{aligned} 3x + y &= 10 \\ 3x + 4 &= 10 \\ -4 \quad &\quad -4 \\ 3x &= 6 \\ \frac{3x}{3} &= \frac{6}{3} \\ \boxed{x = 2} \end{aligned}$$

Solution: (2, 4)

Example Two: *Multiply equations, then add or subtract to eliminate a variable.*

ADD!

$$\begin{aligned} 3(3x + 2y &= -26) \leftarrow \text{Multiply by 3} \\ 2x - 6y &= -10 \\ + \quad 9x + 6y &= -78 \\ \hline 11x &= -88 \\ \frac{11x}{11} &= \frac{-88}{11} \\ \boxed{x = -8} \end{aligned}$$

Substitute:

$$\begin{aligned} 2x - 6y &= -10 \\ 2(-8) - 6y &= -10 \\ -16 - 6y &= -10 \\ +16 \quad &\quad +16 \\ -6y &= 6 \\ \frac{-6y}{-6} &= \frac{6}{-6} \\ \boxed{y = -1} \end{aligned}$$

Solution: (-8, -1)

Subtract!

$$\begin{aligned} 2(5x + 3y &= 2) \leftarrow \text{Multiply by 2} \\ 3(4x + 2y &= 10) \leftarrow \text{Multiply by 3} \\ 10x + 6y &= 4 \\ - \quad 12x + 6y &= 30 \\ \hline -2x &= -26 \\ \frac{-2x}{-2} &= \frac{-26}{-2} \\ \boxed{x = 13} \end{aligned}$$

Substitute:

$$\begin{aligned} 5x + 3y &= 2 \\ 5(13) + 3y &= 2 \\ 65 + 3y &= 2 \\ -65 \quad &\quad -65 \\ 3y &= -63 \\ \frac{3y}{3} &= \frac{-63}{3} \\ \boxed{y = -21} \end{aligned}$$

Solution: (13, -21)